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LETTER TO THE EDITOR

Crossover from mesoscopic to classical proximity effects, induced by particle–hole symmetry breaking in Andreev interferometers

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Abstract. When two superconducting contacts are made on either side of a mesoscopic normal wire, the electrical conductance is a periodic function of the phase difference between the superconductors. For this structure, the oscillation at zero voltage and zero temperature is a small mesoscopic effect, with an amplitude of order e^2/h. In contrast, we predict that a finite bias voltage V will induce giant oscillations associated with the classical proximity effect. These are a finite fraction of the overall conductance, exhibit a maximum when eV equals the Thouless energy, and decrease at higher voltages. This effect may account for the large-amplitude oscillations measured in recent experiments by Petrashov *et al*.

Recent studies of normal–superconducting (N–S) nanostructures have highlighted a variety of new phenomena associated with phase-coherent transport in the presence of Andreev scattering [1–10]. For systems consisting of a superconductor in contact with a semiconductor (Sm), the transmittance of the Schottky barrier (I) at the S–Sm interface depends strongly on the carrier concentration in the semiconductor, which in turn can be varied by doping or by tuning a gate voltage. Experiments on such structures [1–5] have established that the subgap conductance of SIN junctions with a sufficiently high barrier is not small at low temperatures T. Moreover a peak in the conductance arises at zero bias, the magnitude of which increases with decreasing T and becomes comparable with the conductance in the normal state. An explanation for these phenomena was provided by several authors [12–14, 20] starting from Zaitsev who developed a microscopic theory of the subgap conductance [11]. Using microscopic equations for matrix Green functions and associated boundary conditions, he calculated the differential conductance Sd for short contacts S/N1/N2 (where a slash indicates an interface with a finite transmittance) and showed that the zero-temperature conductance has a peak at zero bias (the zero-bias anomaly). Later this theory was developed for contacts of different types and dimensions [12, 13, 23] and alternative approaches suggested [14, 15, 20].

More recently, following earlier theoretical papers [16–20] on disordered transport in the presence of two superconducting contacts, several new experiments [6–9] have probed the phase-coherent nature of Andreev scattering. In these experiments, the difference $\phi = \phi_1 - \phi_2$ between the superconducting order parameter phases ϕ_1, ϕ_2 is controlled by some external means and the oscillatory dependence of the conductance on ϕ is measured. Prior to these experiments, it was predicted [16, 17] that for a diffusive system of size L and diffusion coefficient D, in the high-temperature regime $T > T^*$, where $k_B T^* = hD/L^2$, the ensemble-averaged conductance should be a periodic function of ϕ, with fundamental period of π. In contrast at low temperatures $T < T^*$, it was predicted [18] that the ensemble-averaged conductance should have a fundamental period of 2π. The latter prediction was confirmed by detailed numerical simulations [19] and by further theoretical work [20]. Furthermore, in all experiments to date [6–9], the observed fundamental periodicity is 2π.

The experimental realization of such Andreev interferometers has stimulated a number of new theoretical papers [21–27, 31], aimed at describing the amplitude and harmonic structure of phase-periodic conductances. For samples with a conductance much greater than e^2/h, weak localization and mesoscopic fluctuations are negligible and the measured conductance is closely approximated by the ensemble-averaged conductance, which can be calculated by quasi-classical methods. On the other hand if the conductance is of order e^2/h, then the ensemble-averaged conductance is of no interest and the former dominate. The language used to describe quasi-classical calculations is that of the classical proximity effect, whereas calculations of mesoscopic fluctuations and weak localization usually emphasize the role of particle–hole interference. Despite the different languages, all of these theories describe different aspects of the same phenomenon, whose existence ultimately arises from the phase-coherent nature of Andreev scattering.

An exciting experimental result is the observation of oscillations with an amplitude greater than e^2/h [6]. These have since been described using quasi-classical Green function techniques [22–24, 26] and numerical simulation [27, 28] and are predicted to be a feature of Andreev phase-gradiometers, formed from single N–S contacts [25]. A related phenomenon of giant backscattering has also been discussed [29]. These recent developments suggest that studies of the crossover from mesoscopic to proximity effects in S/N structures will prove to be a key testing ground for current theories. In this letter, we examine the effect of increasing the bias voltage in the interferometer of figure 1. Remarkably, we predict that at zero voltage only small oscillations are present, whereas at finite voltages large oscillations arise, due to the proximity effect. These giant oscillations, which arise through particle–hole symmetry breaking, exhibit a maximum amplitude at a voltage of $V^* = k_B T^*/e$ and decrease as the voltage is further increased. Recently it was noted that quasi-classical theory predicts that at zero temperature and zero voltage, the amplitude of oscillation in the experiments of [6] should be zero, and therefore an explanation based on a thermal effect was proposed [30]. Based on the results outlined below, we propose that particle–hole symmetry breaking due to the use of a small but finite bias voltage provides an alternative mechanism for large-amplitude effects, even at zero temperature.

To obtain this prediction, the structure of figure 1 will be analysed using both quasi-classical theory and an exact numerical transfer matrix technique for solving the Bogoliubov–de Gennes equation. If the S/N interface resistance dominates, the quasi-classical calculation of the resistance of the system is obtained by generalizing an earlier theory [11–13] of the subgap conductance in SIN contacts. This focuses on the following component of the current, which in the case of a Josephson SIS junction, reduces to the so-called interference current:

$$I_{int} = -(8R_N)^{-1} \int \mathrm{d}\epsilon \; f_V(\epsilon)(F^R + F^A)(F_S^R + F_S^A). \qquad (1)$$

In this expression R_N is the resistance of a SIN contact in the normal state; $f_V(\epsilon) = (\tanh(\epsilon + eV)\beta - \tanh(\epsilon - eV)\beta)/2$ is the difference of the distribution functions; $\beta = (2T)^{-1}$; and $F_S^{R(A)}$ and $F^{R(A)}$ are the condensate retarded (advanced) Green functions in the superconducting and normal electrodes, respectively. In the latter case they differ from zero due to the proximity effect. In the case of a planar junction and a weak proximity effect, if

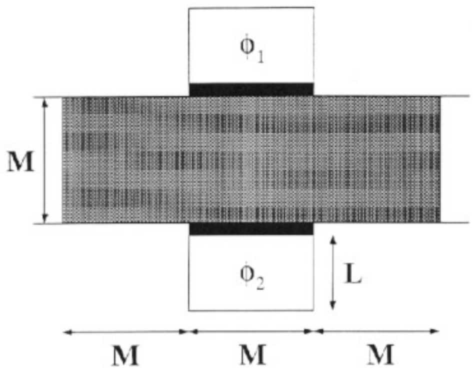

Figure 1. An Andreev interferometer formed from two superconductors with order parameter phases ϕ_1, ϕ_2, in contact with a diffusive normal conductor (shown shaded) via tunnel barriers (shown in black). The current flows from left to right through the normal conductor.

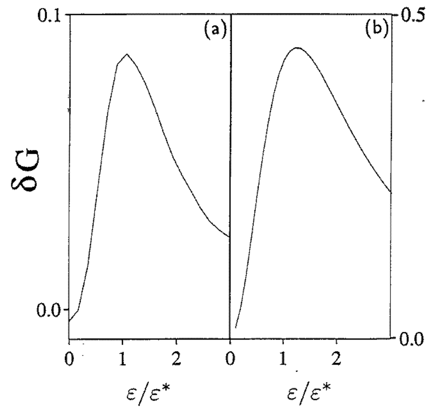

Figure 2. Numerical results (a) and quasi-classical results (b) for the the amplitude $\delta G = [\langle G(\pi)\rangle - \langle G(0)\rangle]/\langle G(0)\rangle$ as a function of the scaled energy ϵ/ϵ^*, where ϵ^* is the Thouless energy.

D, σ, d are the diffusion coefficient, specific conductivity and thickness of the normal film, respectively and R_N is the interface resistance per unit area in the normal state, they are equal to

$$F^{R(A)} = \pm \frac{\mathrm{i}\epsilon_N}{\epsilon \pm \mathrm{i}\gamma} F_S^{R(A)} \tag{2}$$

where $\epsilon_N = D/(2\sigma d R_N)$ and $F_S^{R(A)} = \Delta/((\epsilon \pm \mathrm{i}0)^2 - \Delta^2)^{1/2}$ are the retarded (advanced) Green functions in the S film. The restriction to a weak proximity effect ensures that the latter are unperturbed by the proximity effect itself. In equation (2), γ is the depairing rate in the N film, determined by paramagnetic impurities, inelastic scattering processes and an applied magnetic field $\boldsymbol{H}$. It follows from equations (1) and (2) that the interference current I_{int} in a SIN contact is non-zero only in the second order of the barrier transmittance (i.e. it is proportional to R_N^{-2}) and it is small if the energy or γ is large. The product $(F^R + F^A)(F_S^R + F_S^A)$ differs from zero only in the energy interval $|\epsilon| < \Delta$, and is related to the Andreev scattering processes. If the energy or γ is small enough, this product is not small and the conductance related to I_{int} can be comparable with the conductance in the normal state.

In the case of an interferometer comprising two superconductors contacting the normal film and separated by a distance of less than $L_T = D/\sqrt{T^2 + \gamma^2}$ one should use, instead of equation (2), the more general formula written in a matrix form

$$\hat{F}^{R(A)} = \pm \frac{\mathrm{i}\epsilon}{\epsilon \pm \mathrm{i}\gamma} (\hat{F}_{S1}^{R(A)} + \hat{F}_{S2}^{R(A)}) \tag{3}$$

and calculate the trace of the product $(\hat{F}^R + \hat{F}^A)((\hat{F}_{S1}^R + \hat{F}_{S1}^A) + (\hat{F}_{S2}^R + \hat{F}_{S2}^A))$, where $\hat{F}_{S1}^{R(A)} = F_{S1}^{R(A)}\,\mathrm{i}\hat{\sigma}_y$, $\hat{F}_{S2}^{R(A)} = F_{S2}^{R(A)}(\mathrm{i}\hat{\sigma}_y \cos\phi + \mathrm{i}\hat{\sigma}_x \sin\phi)$. In this case, the conductance of the system will depend on $\cos\phi$, where ϕ is the phase difference between the superconductors, and by measuring the amplitude of these oscillations one can determine the subgap conductance S_{sg}, as suggested by Hekking and Nazarov [20].

If the resistance of the S/N interface is small, the main contribution to the total resistance of the S/N system is caused by the normal film. As mentioned above, the condensate functions $F^{R(A)}$ are induced in the N film due to the proximity effect (we note that $\Delta_N \cong 0$ if the electron–phonon coupling constant in the normal film is negligible) and the resistance of the N film is changed as compared to its value in the normal state (i.e. above critical temperature of the superconductor) [12]. This deviation is caused by a change in the distribution function, compared with its form in the normal state. Indeed, the spatial dependence of the distribution function $f(\epsilon)$ describing the branch imbalance and the electric potential $V(x)$ is described by the equation (outside of the overlapping S/N region) [11–13]

$$\partial_x (1 - G^R G^A - F^R F^A)\ \partial_x f(x, \epsilon) = 0. \tag{4}$$

Integrating this equation once, we obtain

$$[1 - G^R G^A - F^R F^A]\ \partial_x f(x, \epsilon = 2J(\epsilon) \tag{5}$$

where the integration constant $J(\epsilon)$ determines the current in the N film

$$I = (\sigma d/4) \int \mathrm{d}\epsilon\ J(\epsilon). \tag{6}$$

In the normal state, the expression in the square brackets in equation (5) equals 2, because $G^R = -G^A = 1$ and $F^{R(A)} = 0$. Below T_c the condensate functions $F^{R(A)}$ decay exponentially along the N film over a correlation length $L_\epsilon = D/\sqrt{\epsilon^2 + \gamma^2}$. The functions $G^{R(A)}$ and $F^{R(A)}$ are connected by the normalization condition

$$(G^{R(A)})^2 - (F^{R(A)})^2 = 1. \tag{7}$$

Therefore the expression in the square brackets in equation (5) can be rewritten in the form

$$[(G^R - G^A)^2 - (F^R - F^A)^2]/2 = 2[(\mathrm{Re}\, G^R)^2 - (\mathrm{Im}\, F^R)^2].$$

If the condensate functions are small (i.e. the proximity effect is weak), then using equation (7) one can rewrite equation (5) in the form

$$[1 - (F^R - F^A)^2/4]\ \partial_x f(x, \epsilon) = J(\epsilon). \tag{8}$$

The condensate functions in equation (8) lead to a change in a spatial dependence of the distribution function $f(x, \epsilon)$ when compared with its dependence in the normal state and therefore to a change in the conductance of the normal film.

For the system shown in figure 1, assuming that the condensate functions in the N film are small, i.e. $|F^{R(A)}| \ll 1$, one can find a relation between the current in the system and a voltage drop between the end-points, and hence an expression for the zero-temperature, impurity-averaged, phase-periodic conductance $\langle G(\phi)\rangle$. For that purpose it is necessary to solve a linear equation for $F^{R(A)}(x)$ as was done, for example, in [13, 26], and to calculate the current using equations (6) and (8). The result for the amplitude of oscillation

$$\delta G = [\langle G(0)\rangle - \langle G(\pi)\rangle]/\langle G(0)\rangle$$

is

$$\delta G = 2(k_0^2 L_s)^2 L^2 \chi(\epsilon)/(1 - \epsilon^2/\Delta^2)$$

where $k_0 = [G_{\mathrm{barrier}}/(G_N M L)]^{1/2}$. Writing $k_\epsilon = (\epsilon/2D)^{1/2}(1 - \mathrm{i})$, $\theta = k_\epsilon L = \theta_1 - \mathrm{i}\theta_2$ and $r = [\theta \coth(\theta)]^{-1}$, yields

$$\chi(\epsilon) = \frac{|r|^2}{8}\left[\frac{\sinh(4\theta_1)/\theta_1 - \sin(4\theta_2)/\theta_2}{\cosh(2\theta_1) - \cos(2\theta_2)} - 4\mathrm{Re}\left\{\coth\theta^*\left(\frac{\sinh^2\theta_1}{\theta_1} - \mathrm{i}\frac{\sin^2\theta_2}{\theta_2}\right)\right\}\right] - \frac{1}{4}\mathrm{Re}\left\{\frac{r^2}{\sinh^2\theta}\left(\frac{\sinh 2\theta}{2\theta} - 1\right)\right\}. \tag{9}$$

Figure 2(b) shows this function plotted against ϵ/ϵ^*. At zero temperature $\delta G(V)$ is given by the expression (9) with ϵ replaced by eV. In order to obtain $\delta G(V)$ at finite temperatures, equation (9) with the weight factor $[1/\cosh^2(\epsilon + eV) + 1/\cosh^2(\epsilon - eV)]/2$ must be integrated over all energies. The amplitude of oscillation vanishes at $\epsilon = eV = 0$, rises to a maximum at $\epsilon/\epsilon^* = 1$ and decreases at higher voltages ($\epsilon^* = D/L^2$).

As an independent check of this behaviour, figure 2(a) shows the result of an exact numerical solution of the Bogoliubov–de Gennes equation, as outlined in [21]. This uses a transfer matrix technique to compute the scattering matrix of a two-dimensional tight-binding realization of figure 1, with diagonal disorder. In conjunction with [18], this technique [19] provided the first prediction that the fundamental periodicity of the ensemble-averaged conductance is 2π, in contrast with the π-periodic effect of [16, 17]. In the simulations used to obtain figure 2(a), the system dimensions were $M = 15$, $L = 10$ sites and $|\Delta|/\epsilon_F = 0.1$, where ϵ_F is the Fermi energy. The Thouless energy ϵ^* was obtained by carrying out a separate simulation on the diffusive, shaded region alone and determining the normal-state conductance G_N and normal-state density of states per site $N(0)$. In terms of these quantities, the Einstein relation yields $\epsilon^* = (h/2e^2)G_N/(N(0)L_{diff}M)$, where $L_{diff} = 3M$ is the length of the diffusive region. For the simulation of figure 2(a), $\epsilon_*/\epsilon_F = 0.0025$. The numerical results of figure 2(a) were obtained by ensemble averaging over a large number of realizations of the disorder and clearly exhibit the same qualitative features of the quasi-classical prediction. The latter predicts that δG is positive for all finite ϵ and therefore $G(\phi)$ exhibits a zero-phase maximum. The numerical work agrees with this prediction, except for for small ϵ, where the quasi-classical prediction vanishes, but the numerical work predicts a small negative value for δG [32]. In this limit however, fluctuations are important and the mean conductance is no longer a relevant quantity. For a given sample [19, 21], it is known that the precise form of the remaining mesoscopic contribution to $G(\phi)$ depends on the impurity configuration.

The above effect arises because at zero ϵ, particle–hole symmetry ensures that the quantity $k_\epsilon L$, which characterizes the difference between the accumulated phase of particle and hole wavefunctions, vanishes at zero bias. This symmetry is broken at finite energies, and therefore for structures such as that of [6], with a large value of $G(0)$, the amplitude is extremely sensitive to the applied voltage. This effect may prove to be a useful experimental tool, because by controlling the bias voltage in the structure of figure 1, it should be possible to study the crossover from the mesoscopic to the classical regime. We note also that the deviation of local conductivity of the system from its value in the normal state $\delta G(x)$ (x is the distance from the centre of the channel) decays with x nonexponentially. At $T \gg \epsilon^*$ one can obtain $\delta G(x) \propto (1 + \cos\phi)(1 - x/L)^2$. This nonexponential dependence is caused by a contribution of the anomalous terms $F^R F^A$ (see (8)) which decays with increasing energy over a characteristic value ϵ^*.

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and is due to a combination of finite-size restrictions on the numerical work and various approximations inherent in the quasi-classical theory.